Nonlinear modes of the nonlinear Schrödinger equation with complex periodic potential

G.L.Alfimov[1,3], P.P.Kizin[1], V.V.Konotop[2], D.A.Zezyulin[2]

[1] National Research University of Electronic Technology "MIET", Moscow, 124498, Russia
[2] Centro de Física Teórica e Computacional, Faculdade de Ciências, Universidade de Lisboa, Instituto para Investigação Interdisciplinar, Avenida Professor Gama Pinto 2, Lisboa 1649-003, Portugal
[3] email: *galfimov@yahoo.com*

Abstract:

In the talk, we address the problem of description of nonlinear states for the nonlinear Schrödinger (NLS) equation with a complex periodic potential $W(x) = U(x) + iV(x)$ and repulsive (defocusing) nonlinearity.

$$i\Psi_t = -\Psi_{xx} + W(x)\Psi + |\Psi|^2\Psi. \tag{1}$$

Nonlinear states for Eq.(1) are of the form $\Psi(x,t) = \rho(x)\exp\{-i(\omega t + \phi(x))\}$. We study the dynamics of Poincaré map T generated by the system of ODE in 3D space (ρ, ρ_x, θ), $\theta = \phi_x$, for fixed value of the frequency ω. An important peculiarity of the system is that for a wide class of the potentials $W(x)$ the "most part" of its solutions collapse (i.e. tend to infinity at some finite value $x_0 \in \mathbf{R}$). Thus, the dynamics generated by the map T can be described in terms of T-iterations of specific sets $\mathcal{U}_\pi^+$ and $\mathcal{U}_\pi^-$. These sets consist of the points from (ρ, ρ_x, θ) which have T-image (the set $\mathcal{U}_\pi^+$) or T-pre-image (the set $\mathcal{U}_\pi^-$) and must be found numerically. The main attention is focused on the cases when $W(x)$ is real potential, $\mathcal{PT}$-symmetric potential, and more general case, including "nearly-$\mathcal{PT}$"- symmetric potentials.